15 Structural equation modeling and the analysis of long-term monitoring data

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Introduction

The analysis of long-term monitoring data is increasingly important; not only for the discovery and documentation of changes in environmental systems, but also as an enterprise whose fruits validate the allocation of effort and scarce funds to monitoring. In simple terms, we may distinguish between the detection of change in some ecosystem attribute versus the investigation of causes and consequences associated with that change. The statistical framework known as structural equation modeling (SEM) can contribute to both detection of changes and the search for causes. This chapter summarizes some of the capabilities of SEM and shows a few ways it can be used to model temporal change. Because of its ability to test hypotheses about whether rates of change are zero or nonzero, it can be used for change detection with repeated-measures data. As more of the capabilities of SEM are presented, its capacity for evaluating causal networks is highlighted. Here is where its potential for making a unique contribution to the analysis of long-term monitoring data is revealed. Thus, one's primary motivation for using SEM with monitoring data will be to investigate hypotheses about what factors may be driving change (Box 15.1).

In this chapter it will be necessary to first introduce notation to describe the elements of structural equation models (SE models) so as to permit an unambiguous presentation of their various forms. The first part of the chapter works through the fundamental features of models of increasing complexity, while the second part of the chapter illustrates several of these possibilities using a real example.

The elements of structural equation models

Structural equation models can often be most easily introduced by contrasting their form with the familiar univariate model

$$y = \boldsymbol{\beta}\mathbf{x} + \varepsilon \tag{15.1}$$

where y is a single response variable, $\mathbf{x}$ is a vector of predictor variables, $\boldsymbol{\beta}$ is a vector of prediction coefficients, and ε is an error variable that represents the effects of all

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Box 15.1 Take-home messages for program managers

Monitoring programs seek to detect change if it is occurring, to infer the causes behind observed changes, and to evaluate the implications of these changes. The methodology presented in this chapter, structural equation modeling (SEM), has characteristics that make it useful for all three of these goals in the face of common difficulties such as short or incomplete monitoring records and the challenge of discerning the relative importance of multiple factors operating simultaneously. SEM is a flexible methodology for building and testing models that seek to describe processes and relationships that regulate the complex behavior of an ecological system. SEM applications range from simple ones readily implemented with off-the-shelf software to more complex ones that require special programming and experience. For example, using a relatively simple latent trajectory model to consider how multiple sites change over time, it is possible to estimate linear trends with as few as three time periods and nonlinear trends with four or more. It is also possible to consider the divergent trends that may occur for different sites and the network of contributing factors that cause the divergence. For example, in this chapter we demonstrated the use of SEM to examine post-fire temporal trajectories in plant species richness for 88 study plots in a shrub ecosystem, incorporating a network of direct and indirect relationships among plot-level richness, cover, soil characteristic, and fire severity as well as area-wide yearly precipitation. Groups of sites can be formally compared (e.g. those exposed to some factor such as a management treatment and those not exposed) and strategies for handling missing data are available. The SEM process should be guided by careful thought to identify specific questions about potentially important processes and relationships in the system being monitored. The investment in SEM can lead to valuable insights into how and why changes may be occurring and how various factors are affecting the system – information that is essential for guiding how managers respond to these changes.

influences on y other than those in $\mathbf{x}$ (see Chapter 11 for further review). This model can be generalized to the multivariate response case (e.g. multivariate regression or MANOVA) by allowing y and ε to be vectors $\mathbf{y}$ and $\boldsymbol{\varepsilon}$ of variables and error terms.

For SEMs, in contrast, the equation for relations among observed variables is

$$\mathbf{y} = \mathbf{B}\mathbf{y} + \boldsymbol{\Gamma}\mathbf{x} + \boldsymbol{\zeta}, \tag{15.2}$$

where $\mathbf{y}$ is a vector of endogenous variables, $\mathbf{x}$ a vector of exogenous variables, $\mathbf{B}$ and $\boldsymbol{\Gamma}$ are coefficient matrices, and $\boldsymbol{\zeta}$ is a vector of error variables with non-zero elements for each non-zero element in the $\mathbf{y}$ vector on the left side of the equation. The term "endogenous" refers to variables in a network of relations that have arrows pointing to them (i.e. they are response variables somewhere in the model – e.g. variables b, c, and d in Fig. 15.1a–c are all endogenous). The term "exogenous" refers to variables in a network that have no arrows pointing to them (i.e. they only serve as predictors and are not explained by other variables in the model – e.g. variable "a" in Fig. 15.1 is

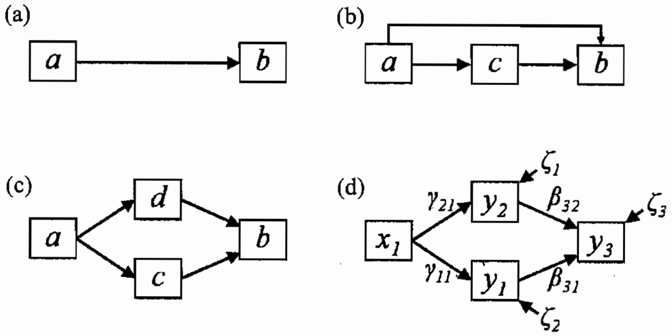

Figure 15.1 Some graphical models discussed in the text: (a) simple regression, (b) path model with direct and indirect pathways explaining the covariance between a and b, (c) path model where total relationship between a and b is explained by the mediating factors c and d, and (d) statistical notation for model in C.

exogenous). The model in Equation (15.2) is sometimes referred to as the econometric model. It is worth noting that SEMs need not be either linear or Gaussian, although often such assumptions are employed.

The inclusion of the term $\mathbf{By}$ in Equation (15.2) has rather profound implications from the standpoint of the statistical models and scientific hypotheses that can be specified and evaluated. In the univariate model [Equation (15.1)] or its multivariate extensions, the reliance on $\boldsymbol{\beta}\mathbf{x}$ to convey the processes influencing y permits only a consideration of direct effects and strongly constrains possible causal interpretations. The combination of terms $\mathbf{By} + \boldsymbol{\Gamma}\mathbf{x}$ in the SEM permits the representation of a network of direct and indirect effects. This equational framework for representing (potentially complex) networks of causal relations creates a need for a graphical means of representation, which is why graphical modeling and path modeling were invented simultaneously, though the term "graphical modeling" is a recent development.

Wright (1921) first illustrated the decomposition of the net relationship between a cause and an effect into multiple causal pathways including indirect pathways mediated by intervening variables. Wright immediately recognized the need for a graphical representation to accompany the analysis of networks of relationships and developed a system for graphical modeling. As Pearl (1998) has argued, the use of a graphical modeling system to address causal networks comes not only from the need to represent the implications of systems of equations, but also from the fact that the "=" sign in equations only describes mathematical equivalence and is potentially ambiguous with regard to the flow of causation, while the elements of graphical mathematics, such as $\rightarrow$, permit an explicit statement of causal direction (see also Shipley 2000).

Figure 15.1 provides the graphical representation of some simple models. In Fig. 15.1a, we see a directional relationship between two variables, a and b. The single-headed arrow from a to b implies a flow of causation, although some will prefer to think of this as conditional dependence. With an error term for b implied but not shown, the model in Fig. 15.1a corresponds with a simple regression. From a Wrightian perspective, we might ask the scientific question, "To what degree can the dependence of b on a be explained by the effect of a on an intermediary cause c?" This question can be represented in graphical modeling terms as shown in Fig. 15.1b. Here we have specified the possibility that part of the net effect of a on b is propagated through c, while there also exists an independent mechanism whereby a influences b independent of c (the direct path). We may proceed further in our investigation of the causes behind the dependence of b on a and ask whether the remaining direct effect in Fig. 15.1b can be explained by a second intermediary cause d. Such a result could be represented by Fig. 15.1c, which is a model in which variables a and b are considered to be "conditionally independent given c and d" ($a \perp b | c, d$). Mechanistically, we would infer that the effects of a on b are mediated by effects that a has on c and d. Representing variables a–d in terms of x's and y's (Fig. 15.1d) permits us to present our example causal network as a series of equations in matrix form corresponding to our general representation in Equation (15.2):

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \beta_{31} & \beta_{32} & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} \gamma_{11} \\ \gamma_{21} \\ 0 \end{bmatrix} [x_1] + \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix}. \tag{15.3}$$

Note that additional matrices can allow us to model the variances and covariances among exogenous variables as well as covariances among our ζ_i variables.

The enterprise of SEM, because it is concerned with underlying causes, has long utilized latent variables in models. Latent variables can be thought of as hypothesized influences, ones for which we do not have direct measurements but whose influences on measured parts of the system are conspicuous. The classic early example of a latent variable was human intelligence, which was, and remains, an attribute whose properties can only be assessed indirectly, through correlated responses. There are now numerous examples of latent variable ecological models (Grace 2006) involving concepts such as animal body size and life history. In this chapter we also use latent variables to represent random slopes and intercepts for the trajectories over time of individual observation units in sample populations. As we outline in the next paragraph, latent variables can be represented in structural equation models by generalizing the equational framework. Including latent variables in models creates a number of requirements for the estimation process, though ones that can generally be satisfied using procedures developed for this purpose. A further discussion of latent variables and how they are used in SEM can be found in Bollen (2002). Chapters 19 and 20 in this volume also incorporate latent variables as part of their hierarchical modeling approaches.

The latent-variable formulation of SE models is represented by an elaboration of Equation (15.2) into three equations. The relationships among latent variables are specified as

$$\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta}, \tag{15.4}$$

where $\boldsymbol{\eta}$ is a vector of latent responses, $\boldsymbol{\xi}$ is a vector of latent predictors, $\mathbf{B}$ and $\boldsymbol{\Gamma}$ are now matrices containing latent variable covariances (as well as their variances) and $\boldsymbol{\zeta}$ is a vector of errors for the endogenous latent variables. Analogous to the observed-variable

SE model, correlations among the errors of latent endogenous and exogenous variables are specified by matrices (respectively labeled $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$). The two remaining equations

$$\mathbf{x} = \boldsymbol{\Lambda}_x \boldsymbol{\xi} + \boldsymbol{\delta} \tag{15.5}$$

and

$$\mathbf{y} = \boldsymbol{\Lambda}_y \boldsymbol{\eta} + \boldsymbol{\varepsilon} \tag{15.6}$$

allow the specification of relationships between latent and observed variables, where the $\boldsymbol{\Lambda}$s are loadings connecting latent with observed variables. We will see in the next section some of the ways that latent variables can be used in SEMs when we use them to help us model longitudinal data.

Before moving to a discussion of how SEMs can be used to represent the processes associated with temporal dynamics, it is important we mention that the SEM enterprise places the specification and estimation of SEMs into a workflow process that is designed to allow us to learn about the causal processes operating in complex systems. The broad enterprise of SEM includes (i) theory formalization, (ii) development of general hypotheses, (iii) specification of theory-based statistical models, (iv) estimation of parameters, (v) evaluation of models, and (vi) synthesis of results. Descriptions of this broader enterprise as it relates to ecological systems can be found in Grace (2006) and Grace *et al.* (2010), but are also briefly presented later in this chapter.

Modeling temporal dynamics using structural equation models

Modern SEM owes much of its development to its adoption and use in the human sciences including psychology, sociology, economics, education, and marketing. While the environmental sciences are now initiating and implementing many large monitoring programs, the human sciences have been collecting such data for a great deal longer and with a much greater commitment of resources. As a result, there is likely much we can learn about analyzing and modeling long-term environmental data from the SEM tradition in the human sciences.

SEMs have been applied to long-term ecological data in a few cases. A simple approach sometimes used has been to summarize net system responses. For example, Clark *et al.* (2007) examined factors controlling net diversity responses to experimental fertilization treatments in herbaceous ecosystems. Larson and Grace (2004) examined the factors affecting net proportional changes in exotic plant populations subject to biocontrol additions. These studies model temporal change by looking at spatial variations in net changes over time; this simple approach can be useful when interest in temporal change can be reduced to some summary measures (final time minus initial time). As this chapter illustrates, SEM can also be used to represent much more complex temporal models.

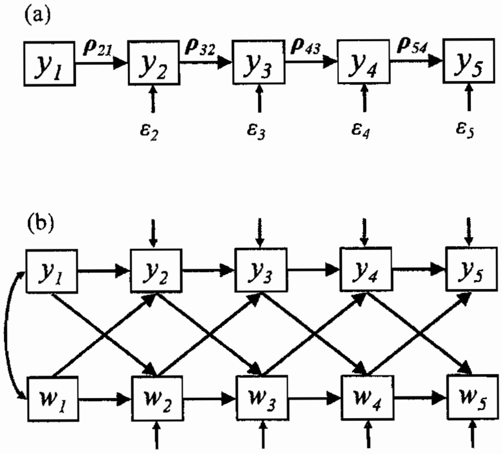

Figure 15.2 Two examples of autoregressive models. In both cases, the subscripts refer to time intervals (i.e. y_3 represents the measurement of variable y at time 3): (a) a simple autoregressive chain, and (b) an autoregressive cross-lagged model involving a response y and a covariate w.

The autoregressive model (ARM)

For the analysis of temporal dynamics using structural equations, two of the main approaches that have been used are (i) autoregressive models (ARMs), and (ii) latent trajectory models (also known as latent growth curve models). The first of these, the autoregressive model, utilizes a time-step equation of the form

$$y_{it} = \alpha_t + \rho_{t,t-1} y_{i,t-1} + \varepsilon_{it}, \tag{15.7}$$

where y_{it} and $y_{i,t-1}$ represent responses by the ith individual (e.g. an organism or sample site) at time t and $t - 1$ respectively, α_t is the time-specific intercept, $\rho_{t,t-1}$ is the autoregressive coefficient, and ε_{it} is the error for individual i at time t (assumed to have mean of zero, to be free of autocorrelation, and to be uncorrelated with y_{it}). For the autoregressive model, the values of y observed at any time are predictable from its values the previous time (or some earlier time periods) plus coefficients predicting changes between times based on either intrinsic or extrinsic factors. Such a model approach can be used to estimate, for example, the fidelity of a population of animals to a particular site over time. Larson and Grace (2004) used an autoregressive SE model to estimate the fidelity from one year to the next of biocontrol insects (*Aphthona* spp., flea beetles) living on their exotic host, leafy spurge (*Euphorbia esula* L.).

An equation that specifies how y_{it} depends on covariates is needed if a general model for temporal dynamics is to be developed from the autoregressive equation. A simple hypothesis that can be evaluated is that $\rho_{t,t-1}$ is constant for all time periods. A more elaborate model would be that $\rho_{t,t-1}$ is a function that drives a general shape to the time trajectory, such as a geometric or asymptotic trend. A graphical representation of a simple autoregressive model is shown in Fig. 15.2a. For such models we typically assume errors are uncorrelated; however, autocorrelation among errors is permitted when appropriate. We will illustrate the incorporation of an autoregressive relationship into an SEM later in the chapter.

Autoregressive models can be elaborated to represent interacting trajectories of responses. One such elaboration is the autoregressive cross-lagged model (ARCL) shown in Fig. 15.2b. Such models and their variants can allow us to partition several kinds of interactive processes between ecological entities, such as fidelity from time to time, competitive or facilitative interactions, or interactions between organisms and their resources. Again, in such models various kinds of correlated error structures are permitted when appropriate. The ecological interaction between the plant leafy spurge and its biocontrol insects, mentioned above, is one example where a cross-lagged autoregressive model was used to represent interactions playing out over time. In this case, it was possible to assess whether effects on plants from the insects caused lag responses from the previous year or only immediate impacts from the current year.

The latent trajectory model (LTM)

A second approach to time-change modeling describes a curve or trajectory that can be fit through the repeated observations of individuals. This also is referred to as latent curve or growth curve modeling. The first level equation for the latent trajectory model is

$$y_{it} = \alpha_i + \lambda_t \beta_i + \varepsilon_{it}, \tag{15.8}$$

where y_{it} represents individual responses, α_i describes the across-time intercept for the trajectory of y_i, and ε_{it} is again the error for individual i at time t. In this equation, β_i is the random slope for individual i and λ_t is a device for coding time (and can be either linear or nonlinear depending on the coding). Following the convention of Bollen and Curran (2006), for the linear model the time periods are numbered $0, 1, \ldots, T-1$; therefore, $\lambda_1 = 0$, $\lambda_2 = 1$, and $\lambda_t = T-1$, where $T =$ total number of time periods in the data set.

Because the intercept and slope terms are random effects in Equation (15.8), it holds that there is a second level of equations

$$\alpha_i = \mu_\alpha + \zeta_{\alpha i} \tag{15.9}$$

and

$$\beta_i = \mu_\beta + \zeta_{\beta i} \tag{15.10}$$

where μ_α and μ_β are means for the intercept and slope, respectively, and the ζs are error terms. The combination of these three equations represents a standard hierarchical or multilevel model of the sort commonly used in statistical analysis (e.g. Chapters 7, 11, 12, 19, 20). In this case, Equation (15.8) is referred to as a level 1 or lower-level equation while Equations (15. 9) and (15.10) are level 2 or upper level equations (μ_α and μ_β are sometimes referred to as hyperparameters, although this is a limiting terminology where there are more than two levels in the model). One difference between the use of this multilevel modeling feature in SEM versus conventional mixed models (e.g. Pinheiro and Bates 2000) is that in SEM covariate effects at both levels can be organized in networks, as illustrated later in the chapter. Other major differences have to do with (i) the fact that the covariance procedures in SEM can lead to rejection of our models and the search for specifications more consistent with the data, (ii) the precise control of assumptions about parameters (e.g. variance inequalities, error correlations) in SEM, and (iii) the universal capability for using latent variables to address measurement questions.

There has been significant interest in the use of latent trajectory models within the SEM framework. Meredith and Tisak (1990) first showed that the SEM latent variable modeling system [Equations (15.4)–(15.6)] could be used to represent random-effects trajectory models by setting the loadings for a factor model to a set of time steps. To elaborate, in factor models one or more latent variables are hypothesized to explain the covariances among a set of observed variables. As an example, Sewall Wright (1918) developed a factor model that hypothesized a set of animal body-size developmental factors (latent variables) to explain the patterns of correlations among bone lengths. In typical factor models, the loadings (regression weights linking observed to latent variables) are freely estimated (although sometimes one loading will be set to a value of 1.0 to specify the scale of measurement for the latent variable). This "factor-analytic" architecture is easily represented using the linear structural relations (LISREL) equations (Jöreskog 1973, equations 4–6a) and commonly included in SEMs. The invention of Meredith and Tisak (1990) was to conceptualize the slope of a time trajectory as a latent variable and then set the loadings for its relationship to a sequence of measurements to the time-step values (one can peek ahead to Fig. 15.7 to see a graphical representation of this model showing the loadings). A number of authors have elaborated on this framework and there are now some excellent treatments of latent trajectory modeling from an SEM perspective (Bollen and Curran 2006, Duncan *et al.* 2006).

The unconditional latent trajectory model, unconditional in the sense that it ignores covariates and only describes the shape of the trajectory, can be specified using the measurement model [Equation (15.6)]:

$$\mathbf{y} = \boldsymbol{\Lambda}_{\mathbf{y}}\boldsymbol{\eta} + \boldsymbol{\varepsilon}. \tag{15.11}$$

Here $\mathbf{y}$ describes the vector of repeated measures $(y_0 \ldots y_{T-1})$ while $\boldsymbol{\eta}$ represents the latent factors describing the trajectories. In the simplest case, $\boldsymbol{\eta}$ has two elements

$$\boldsymbol{\eta} = \begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix}, \tag{15.12}$$

the random intercepts (α_i) and slopes (β_i) for each individual in the sample. In this simple case, we require a minimum of three time periods of measurement to be able to identify the estimated slope. For trajectories with more complex shapes, $\boldsymbol{\eta}$ can have more terms and we have the general requirement that there be at least one more time period of measurement than slope parameters (e.g. a quadratic model would require four time periods).

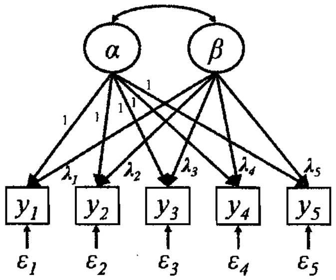

Figure 15.3 Simple latent trajectory model (LTM). In this model the trajectory described by observed measurements of response variable y over five time periods can be explained by an intercept α and slope β. For the linear model, the values for $\lambda_1, \ldots, \lambda_5 = 0, 1, 2, 3$, and 4.

The matrix $\mathbf{\Lambda}$ is a pattern matrix with columns representing the loadings for the intercept and slope terms

$$\mathbf{\Lambda} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ \vdots & \vdots \\ 1 & T-1 \end{pmatrix}. \tag{15.13}$$

It should be noted that while this latent trajectory formulation has the general form of a factor model, the constraints placed on $\mathbf{\Lambda}$ give it a different and more specific purpose (the latent variables in this case serve as intercept and slope). Graphically, the basic latent trajectory model can be represented as in Fig. 15.3.

Nonlinear trajectories

Although there are several different ways we can modify Equation (15.11) to represent nonlinear trajectories, it is also possible to use it for certain situations where temporal changes are curvilinear or where time steps are unequal. First we will describe the latter, simple case, which involves estimating nonlinear factor loadings for the slope in the $\mathbf{\Lambda}$ matrix. In the example given above in Equation (15.13), the column of slope weightings is both linear and "fixed", with values $0, 1, 2, \ldots, T-1$ (note that specifying values for a parameter is often referred to as "fixing" the value of the parameter in SEM parlance). Rather than fixing the values for the slope loadings as shown in Equation (15.13), we can estimate some of the values, as represented in Equation (15.14), where the parameters for the third and subsequent time steps are estimated rather than given a fixed value:

$$\mathbf{\Lambda} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & \lambda_3 \\ \vdots & \vdots \\ 1 & \lambda_T \end{pmatrix}. \tag{15.14}$$

To establish the scale, we can, for example, fix the first loading (λ_1) at a value of 0 and the second loading (λ_2) at a value of 1 so that the first time interval has a unit length. Allowing the loadings for subsequent time periods to be freely estimated permits us to model the empirical data using a nonparametric approach similar to a spline. Rather than specifying the first two loadings, as just described, an alternative approach could be to fix the first and last loadings and allow the two intervening ones to be estimated.

A parametric approach to estimating nonlinear trajectories might be to use a polynomial series to represent slope parameters. To obtain a quadratic model, we expand Equation (15.8):

$$y_{it} = \alpha_i + \lambda_t \beta_{1i} + \lambda_t^2 \beta_{2i} + \varepsilon_{it}. \tag{15.15}$$

The introduced term λ_t^2 is the square of the time loading (for time period 2, it would be a value of 4) and β_{2i} represents the coefficient for the quadratic term for individual i. Thus, we are constructing our quadratic model through the squaring of the loadings instead of the squaring of beta (the slope term), allowing us to remain linear in our estimated parameters while modeling a nonlinear relationship. Using our convention of setting the first loading to 0 and second loading to 1 as in Equation (15.14), we can interpret α_i as the intercept, β_{1i} as the linear slope, and β_{2i} as a measurement of degree of curvature.

Certain additional nonlinear forms of the LTM have been proposed by du Toit and Cudeck (2001) and described in Bollen and Curran (2006, chapter 4, section 4). One nonlinear form of particular merit for modeling asymptotic change is the exponential,

$$y_{it} = \alpha_i + (1 - e^{-\upsilon \lambda_t})\beta_i + \varepsilon_{it}. \tag{15.16}$$

Exponential functions are convenient to use when trajectories tend toward an asymptotic value. In this case, β_i describes the expected total change in y_i while $e^{-\upsilon}$ represents the rate of deceleration of change over time. Typically, υ (upsilon) will be treated as a fixed effect across cases. The specification of the exponential model is similar to that shown in Equation (15.14) except $\lambda_2 = 1 - e^{-\upsilon(1)}$, $\lambda_3 = 1 - e^{-\upsilon(2)}$, and $\lambda_4 = 1 - e^{-\upsilon(3)}$. Additional information about nonlinear approaches to latent trajectory modeling can be found in Ram and Grimm (2007) and Grimm and Ram (2009).

The inclusion of covariates

One strength of the SEM approach is the capacity to include covariates in models in ways that can reflect the mechanistic processes that generate relationships in the data. In simple terms, we can think of the types of covariates that can be included as relating (directly or indirectly) to either the latent intercepts and slopes or to the observed responses themselves. Covariates that affect the slopes and intercepts of latent trajectory models are sometimes referred to as "time-invariant" while those that affect observed responses are commonly "time-varying" (having different values for each time period). In the ecological example that follows, we will include both types for illustrative purposes. Here we describe a few of the equational forms that can be used to represent covariates in SE models.

For covariates that affect the random slopes or intercepts of latent trajectory models, their impacts are not directly on the level 1 equation [Equation (15.8)], but instead, are in the level 2 equations:

$$\alpha_i = \mu_\alpha + \sum \gamma_\alpha \mathbf{x}_i + \zeta_{\alpha i} \tag{15.17}$$

and

$$\beta_i = \mu_\beta + \sum \gamma_\beta \mathbf{x}_i + \zeta_{\beta i}. \tag{15.18}$$

Here, γ_α and γ_β are vectors of coefficients for the covariates in vector $\mathbf{x}_i$ that affect either α_i or β_i.

We also may wish to include additional covariates that have specific time-varying effects. To do so requires expansion of our level 1 equation, which in vector form is

$$\mathbf{y}_{it} = \boldsymbol{\alpha}_i + \boldsymbol{\lambda}_t \boldsymbol{\beta}_i + \boldsymbol{\gamma}_t \mathbf{w}_{it} + \boldsymbol{\varepsilon}_{it}, \tag{15.19}$$

where $\mathbf{w}_{it}$ represents the time-specific covariates and γ_t represents their time-specific effects. To complete our presentation, we recognize that the covariates affecting slopes, intercepts, and observed responses can be allowed to be response variables in additional equations describing the overall causal network (as will be illustrated in Fig. 15.10). All of this can be represented mathematically using our general SEM framework, which is given by Equations (15.4)–(15.6).

The autoregressive latent trajectory model (ALT)

At the beginning of the chapter, we described the fundamental structure of autoregressive models [Equation (15.7), Fig. 15.2)]. Such models are appropriate when we wish to represent the values of response variables at time t as being causally affected by their values at time $t - 1$. Curran and Bollen (2001) and Bollen and Curran (2004) have proposed a framework for merging both kinds of causal architectures (the ARM and the LTM) into a single model, which they call the autoregressive latent trajectory model (ALT).

When combining the latent trajectory structure with the autoregressive structure there is an incompatibility that arises that must be addressed. In the ARM model, the first measure in any chain of observations (e.g. y_1 in Fig. 15.2a as well as y_1 and w_1 in Fig. 15.2b) must be treated as predetermined. By allowing for these initial measures to be predetermined, they have determinant (fixed) values and we avoid the problem of "infinite regress" discussed by Bollen and Curran (2004). The problem of infinite regress comes into play when the measures being analyzed are part of a series of potential measures and our measure y_1 has a preceding value for which we have no estimate. If our y_1 is treated as endogenous in our model, there is a bias that can be propagated down the chain; therefore, autoregressive chains typically begin with the first observation period treated as exogenous. The incompatibility between ARM and LTM becomes apparent when we consider that the initial measures in an LTM are not considered exogenous because they serve as indicators for the estimate of the slope and intercept of the trajectory. To merge the two structures, it is usually appropriate to treat the first time measures as exogenous

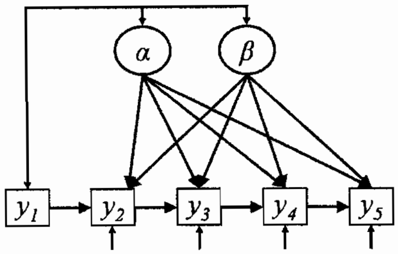

Figure 15.4 Autoregressive latent trajectory model (ALT). Trajectory described by y over time can be explained both by a common intercept α and slope β and also by autoregressive time-specific effects. In this version of the ALT model, the first observation of y is modeled as exogenous and simply correlated with the intercept and slope.

to the latent trajectory as shown in Fig. 15.4. Here, the relationships between the initial measure and the slope and intercept are treated as undirected covariances. The cost of such a model, which can be important in short time series, is that the first time interval (from y_1 to y_2) is not used in the estimation of the slope and intercept. In the exceptional case where the first measurement available is truly the first in the developmental series, we may consider modifying the structure of the ALT model so that the first observation is treated as endogenous to the slope and intercept (α and β). Further discussion of the ALT model can be found in Bollen and Curran (2006).

Issues of model identification

An issue of key importance in statistical modeling is parameter or model identification, which is required so that unique estimates are obtained. While problems with identification can arise for even simple models, complex models of the sort commonly found in SEM have a greater chance of including parameters that are not identified and unique estimates cannot be obtained. Here we provide some general background; a deeper discussion of the issues as they specifically relate to latent trajectory models can be found in Bollen and Curran (2006).

At its simplest level, the issue of identification can be understood from the perspective of the elementary principle of matrix algebra that to estimate the values of two unknown parameters requires two unique equations. So, for the relationship $y = ax_1 + bx_2$, where y, x_1, and x_2 are known, there exists an infinite number of combinations of values for a and b that will satisfy the relationship. However, if we also know that $y = 3ax_1 - bx_2$, we have two unique equations and in principle can derive unique values for a and b that will satisfy the pair of equations. When considering the full set of parameters included in a structural equation model, there is a subset whose values are readily identified from the data. This known set of parameters will typically include the means, variances, and covariances for the observed variables. If the unknown parameters (e.g. path coefficients) are unique functions of the known entities, they are identified. The unknown parameters for the simple LTM described by Equations (15.8)–(15.10) include the means for the latent intercept and latent slope, μ_α and μ_β, as well as the error variances $VAR(\varepsilon_{it})$, $VAR(\zeta_{\alpha i})$, and $VAR(\zeta_{\beta i})$, and the covariance between intercept and slope $COV(\zeta_{\alpha i}, \zeta_{\beta i})$. One simplifying assumption that is made is that the error variances for the individual cases are equal and thus $VAR(\varepsilon_{it}) = VAR(\varepsilon_t)$. There is no assumption made that the error variances are the same over time, however. In the case of the LTM, the loadings λ_t may or may not need to be estimated. In the linear model, all values for λ_t are specified, as shown in Equation (15.13). In models with nonlinear trajectories of fixed form (e.g. the exponential model), all elements of λ_t are again specified. However, in the nonparametric model shown in Equation (15.14), some loadings must be estimated. At a minimum, we should have at least two elements of λ_t specified.

As alluded to above, one condition for model identification that is generally necessary, although not always sufficient, is that there be at least as many known pieces of information as there are unknown parameters to estimate. This principle is helpful in allowing us to determine the number of temporal samples of data that are required for a model to be identified. For the linear model, at least three time points must be included in the dataset for identification. For more complex models, for example one with a polynomial slope structure, at least four time points must be included. Also, for the autoregressive latent trajectory model (Fig. 15.4), four time points will typically be required because the first sample will often be treated as exogenous to the rest of the series and not used to estimate the slope or intercept.

A final important point about identification is that it is possible for non-identification to occur because of empirical matters. For example, a model may be theoretically identified but the estimation process may be unable to obtain unique estimates for the parameters. This can easily happen in the situation where two variables in the data set are nearly perfectly correlated. In such a case, the two variables do not represent independent pieces of information, but instead, act as one. Other situations can produce non-identification problems and in practice they will be called to the investigator's attention by error messages from the software using for the analysis.

Estimation: classical and Bayesian approaches

In what we might call classical SEM (Jöreskog 1973, Bollen 1989), solution procedures have most commonly involved the use of maximum likelihood methods based on the analysis of covariances. A fundamental characteristic of the classical approach to estimating and evaluating the fit of SEMs involves a comparison of the observed variance–covariance matrix (for simplicity referred to simply as the covariance matrix) with the model-implied covariance matrix. This comparison is sometimes represented symbolically as

$$\boldsymbol{\Sigma} = \boldsymbol{\Sigma}(\boldsymbol{\Theta}) \tag{15.20}$$

where $\boldsymbol{\Sigma}$ refers to the population covariance matrix of the measured variables, $\boldsymbol{\Sigma}(\boldsymbol{\Theta})$ refers to the covariance matrix implied by the model as a result of the parameters of the model, and $\boldsymbol{\Theta}$ refers to the full set of model parameters. The sample covariance matrix $\mathbf{S}$ is typically used as our best estimate of $\boldsymbol{\Sigma}$, and $\hat{\boldsymbol{\Theta}}$ refers to the estimated model parameters. There exists a similar relationship between the observed means μ and those implied by the model $\mu(\Theta)$.

Estimation in classical SEM involves the selection of values of $\hat{\Theta}$ so as to make the estimated value of $\Sigma(\Theta)$ as close to S as possible. Maximum likelihood methods are commonly used to arrive at the parameter estimates. A commonly used fitting function is described by the expression

$$F_{\mathrm{ML}} = \ln|\Sigma(\Theta)| - \ln|S| + tr[\Sigma^{-1}(\Theta)S] - p - [\bar{y} - \mu(\Theta)]'\Sigma^{-1}(\Theta)[\bar{y} - \mu(\Theta)], \tag{15.21}$$

where p is the number of observed variables in S and all other symbols are as previously defined. The fitting function has a minimum value of 0, which is obtained whenever the model is fully identified (number of unknown parameters equals number of known variances, covariances, or means). When the model is over-identified, and this will usually be when some pathways are omitted from the model, we can expect some discrepancy between S and $\Sigma(\hat{\Theta})$ and between μ and $\mu(\hat{\Theta})$. Because the fitting function follows a chi-square distribution, the degree of discrepancy between observed and model-implied values can be summarized by a model chi-square and the magnitude of this chi-square can be evaluated for the probability that the observed data are consistent with values implied by the model. The chi-square discrepancy function, then, provides a basis for rejecting a model as inconsistent with the observed data. Such a rejection typically results in the search for a model that is consistent with the data and still theoretically justified. For more information about estimation, refer to Bollen (1989). More about the assessment of model fit and the search for properly specified models is presented below.

Adopting a Bayesian approach to estimating SEMs (Dunson *et al.* 2005, Lee 2007) permits a greater flexibility for modeling. First of all, a great variety of distributional forms for response variables is permitted, although many are permitted in classical modeling procedures as well. Second, equations can be nonlinear in their parameters and still be estimated using the Markov chain Monte Carlo (MCMC) methods often associated with Bayesian applications. Constraints on parameter identification still apply, but opportunities for estimating a greater variety of models open up with a Bayesian approach because the use of prior information can allow for identification of parameters in some cases. A more complete discussion of a Bayesian approach to SEM can be found in Lee (2007).

The treatment of missing data

Missing data are a commonly encountered situation in any analysis. In monitoring studies, missing data are frequently more of a problem than in single-sample studies. A variety of ways of dealing with missing data are used in practice, though not all are equal with regard to either efficiency or bias (Rubin 1976). Chapter 13 outlined methods for dealing with missing data when such "holes" would hinder an objective of producing index counts for a population each year. Here we give a brief presentation on the subject from the perspective of SEMs.

There are many different reasons why data might be missing in a data set. It is even possible that the value for an observation may not be missing initially, but instead, judged to be erroneous or an outlier. When encountering missing data, the reason the data are missing may influence the remedy employed (see also Chapter 13). To review, categories of missing data include:

- "Missing completely at random" (MCAR) refers to when the value of a single observation is missing with equal probability for any observation on any variable from any case in the data set.
- "Missing at random" (MAR) refers to a different situation. Here, we can consider a missing case as MAR even if there is a pattern to the missing data, as long as the mechanism causing missing data does not depend on the unobserved data. That is, MAR assumes that the pattern of missing data is not predicted by the unobserved values of the variables (and thus, that the missing data do not lead to a biased estimate of the predictors of missingness). Let's imagine, for example, that we have data from a sample of a species of grassland birds. Perhaps for some reason the data for a variable are more difficult to obtain reliably from the younger members of the species and this results in a pattern of missing values in the data set (we are more likely to have missing values in younger individuals). If we analyze the data as a single group and bird size or a correlated variable like bird age is used as a predictor in the model, then we classify those missing data as "missing at random" (MAR).
- The condition of "missing not completely at random" (MNAR) is when missing observations predict the pattern of missing values. This condition is one that typically leads to confounding of missing values with other factors and remedies are difficult for such a situation without additional information to use in correcting for the missing values (Schafer and Graham 2002). Remedies involving the MNAR case are beyond the scope of our presentation here.

A number of different approaches are sometimes used for dealing with data that are MCAR or MAR. One of the most common methods for dealing with missing values is casewise deletion, where entire records are removed when one value from those cases is missing. In some situations outside of the SEM context, this strategy would result in entire years of monitoring data being discarded from the analysis (Chapter 13). Apart from these situations, when the sample is large, the casewise deletion of single or a small percentage of cases has modest effects on statistical power. However, when a sizable number of cases are dropped through casewise deletion, this is a highly inefficient method as many data from the non-missing observations for those cases are wasted. It is possible to use pairwise deletion in the estimation of a corrected sample covariance matrix. Again, if only a small percentage of observations are missing, the consequences are not too severe. However, such an approach can be quite unsuitable for situations where there are substantial numbers of missing values, as the resulting matrix is not a coherent representation of the population. Sometimes missing data are addressed by inserting imputed values in place of the missing values (see Chapter 13). In the simplest situation, the mean for a variable is inserted in place of a missing value. Again, when imputed values are inserted for a small number of observations, the consequences are not severe. However, insertion of means has the effect of reducing the variance for a variable when the number of insertions is substantial. Imputation can involve the random selection of a value from the distribution of values for a variable when we wish to avoid inserting means into the dataset. Again, in moderation such an approach has small effects, but is inappropriate when the number of missing values for a variable is substantial. However, it is important to realize that methods such as listwise or pairwise deletion assume data are MCAR and if data are MAR, these deletion methods do not necessarily lead to consistent estimates of model parameters.

Full-information maximum likelihood approach to missing data

In 1996, Arbuckle presented a method for estimating structural equation models in the presence of missing data using all the nonmissing data and ignoring the missing values. This method, which is referred to as full-information maximum likelihood (FIML), is widely regarded as an excellent approach to analysis in the presence of missing data as long as the data can be assumed to be MAR or MCAR. [Note that early SEM references referred to the usual ML, maximum likelihood estimation, without missing data as FIML because it was a ML estimator that made use of all equations and all information in the system to estimate the parameters. Here we are using FIML in the sense proposed by Arbuckle (1996).] The FIML approach retains all the strengths of maximum likelihood while maintaining maximal efficiency of information and minimum bias. The reader is referred to Arbuckle (1996) for a further description of this method.

An overview of the structural equation modeling process

An ambition of the SEM enterprise is learning about causal processes. This involves more than simply the specification of models and estimation of parameters. SEM also involves a workflow process that is designed to advance scientific understanding and the strength of inference (Grace *et al.* 2010). Our presentation of models in the following example is designed to teach the reader, step by step, how to specify an SEM for temporal dynamics. In practice, the experienced SEM practitioner would start with a fully developed model (such as the one shown later in Fig. 15.11) and compare it to a set of related competing models (Box 15.2). Because our presentation in the next section is geared towards teaching technique, here we provide a brief description of the SEM model development and evaluation process.

SEM can be used in three different modes: exploratory, competing models (model comparison), and strictly confirmatory (Jöreskog 1977). Often when practitioners are first applying SEM or when one is approaching a new system or undeveloped topic, the application will be exploratory (we might also call it "model building"). The essential characteristic of an exploratory application is that a large number of models are examined and the final model can be very different from the ones first considered. The example presented in the next section is an illustration of an exploratory approach in that we demonstrate the building of a full model by starting with simple, incomplete models to which additional complexity is added.

Box 15.2 Common challenges: struggling to learn

A major feature of SEM is the comparison between data covariances/correlations and those implied by the models used to evaluate that data. As a result of this feature, the experience of using SEM is one of discovery, where the analyst commonly finds that their models are inconsistent with the data because of some omitted relationship. This discovery process is both highly educational and demanding of patience at the same time. Simple SEM applications are not difficult to learn and apply. However, for more complex models, formal training in SEM or access to an expert in the methodology is commonly needed to navigate the process of discovering suitable models. Those analyzing monitoring data should obtain some training in SEM if they wish to use this methodology. SEM is capable of handling a great number of statistical complexities and difficulties because it is a highly flexible methodological framework rather than a specific analysis technique. Learning how to successfully apply SEM when faced with complex issues requires persistence. It is our plan to continue to provide materials that can help with the learning process.

In mature SEM practice, we generally strive for a competing models approach. In this approach, theoretical ideas are first translated into a conceptual model or "meta-model" (Grace *et al.* 2010) that defines the general ideas being addressed in the analysis. The second step involves the specification of SEMs based on the meta-model. This step requires both consideration of competing hypotheses about mechanisms and also consideration of data characteristics, especially non-Gaussian responses, nonlinear pathways, and hierarchical data structure. Then, specified models should be examined for mathematical suitability to ascertain whether all parameters are potentially identifiable (as discussed above). Once data have been used to estimate values for model parameters, two kinds of model evaluation take place.

The first and most primary form of model evaluation is the assessment of "absolute" fit. As described in the previous section on estimation, it is a characteristic of classical SEM that analyses are conducted on the matrix of covariances. This approach permits a comparison of model-implied to actual covariances. When models include fewer estimable parameters than the known pieces of information in the variance–covariance matrices, we have "model degrees of freedom". Having model degrees of freedom gives us the potential to detect model–data discrepancies. There are a number of different metrics that can be used to assess absolute model fit, though the most fundamental is the model chi-square. As stated previously, the chi-square discrepancy function provides a basis for rejecting a model as inconsistent with the observed data. Thus, it is possible (and common) for models to be found to be "unacceptable" because the data contain relationships not specified in the model. Such a rejection typically results in the search for a model that is consistent with the data and still theoretically justified. Only when we determine that a model has an adequate absolute fit do we trust the parameter estimates. For further discussion see Bollen (1989) or Grace (2006).

Model comparison and selection is, of course, also important. It is possible that we can have more than one model with acceptable fit. This is particularly the case when pathways are included that do not contribute to variance explanation. When comparing models that differ by one parameter (e.g. with and without a path included), competing models are considered to be "nested" and we can use single-degree of freedom tests. These can either be chi-square discrepancy tests as classically used in SEM or they can be conditional independence tests such as the d-sep test (Shipley 2009). It is also possible to use information-theoretic methods to compare models that contain different sets of variables. In this case, the concept of parsimony is given stronger weight in the evaluation. It is beyond our scope to go into the complex topic of model selection in detail here [more can be found in Bollen (1989) and Grace (2006)]. However, we provide a brief description of the model selection/evaluation process used in the example that follows.

In the example presentation below, we first rely on model chi-square to evaluate the overall adequacy of a model (absolute model fit). When a model possesses an associated chi-square that is small (relative to the number of degrees of freedom), this indicates that the cumulative discrepancy between observed covariances and the covariances implied by the model is small and model–data fit is good. In the classic case, we seek models for which the probability associated with a model chi-square given a certain number of degrees of freedom is > 0.05. Note that in null-hypothesis testing, we often declare a parameter to be significant when its associated P-value is *less than* 0.05. However, when assessing model fit, it is important for our confidence in the estimates that the model is not obviously mismatched with our data, so we seek models with low chi-square values (and P-values *greater than* 0.05).

One limitation to the use of the model chi-square is that as statistical power goes up along with the number of cases in a sample, our ability to detect small discrepancies increases. Generally this effect is non-problematic in situations where the sample size is less than, say, 100 or 200. In our example, the number of cases in the data set is 88, so we are comfortable relying on the model chi-square. In situations where there are a large number of cases in a sample, the SEM practitioner will often rely on sample-size-adjusted measures, such as the root mean square of approximation or proportionalized measures of discrepancy (e.g. comparative fit index).

In addition to examining overall model fit, the presentation below relies on single-degree-of-freedom chi-square tests to evaluate hypotheses about individual parameters. The single-degree-of-freedom chi-square difference of 3.84 is often used as a cut-off for declaring two models distinguishable, as the probability of obtaining a value larger than 3.84 from a chi-square distribution with 1 df is < 0.05. So, for example, if we relax a constraint in a model (perhaps we have been omitting a pathway, which effectively sets its value at 0) and now we examine a second model where we estimate that parameter, a drop in model chi-square greater than 3.84 indicates the second model is distinguishably better than the first.

It is worth mentioning that many different bases for model comparison have been offered, both from within SEM circles and from without. Generally, it is the philosophy of SEM to rely on multiple metrics when making a decision about the significance of a parameter (one might use parameter P-values to guide the conduct of chi-square tests, for example). The good news is that it has been shown recently (Murtaugh 2009) that there is a strong convergence between model comparison methods in some situations, which provides reassurance for the multi-metric approach. It is our view that "the decision problem" of statistics (is a parameter to be treated as significant or not?) ultimately requires both evidence and professional judgment, particularly when effect sizes are of interest. This view is particularly relevant to SEM practice where we are theory-oriented rather than null hypothesis-oriented.

To continue our description of the SEM workflow process, further evaluation of conclusions is recommended through the process of sequential learning. Model results are used to inform the theoretical ideas that are the basis for the next round of modeling. A key motivation for followup studies in causal modeling is to test hypothesized explanations for pathways through the evaluation of mediator variables (MacKinnon 2008). As an example, building on the findings from an initial SEM analysis of fire effects on shrublands (Grace and Keeley 2006), Keeley *et al.* (2008) used data from another set of fires to examine more complex SEMs that evaluated the roles of additional mechanisms influencing shrubland recovery. As this is meant to illustrate, SEM philosophy encourages subsequent studies to strengthen our basis for inferring causal interpretations.

An ecological example

Background

In the fall of 1993, major fires burned through southern California, USA, during a short period of time, representing a large-scale disturbance for the region's natural ecosystems. Keeley *et al.* (2005) established one 1000-m^2 plot in each of 90 sample sites in burned areas and began sampling in spring of the first postfire year, continuing for a total of 5 years. Objectives of the study included (i) to determine the degree of recovery of vegetation; (ii) to test certain theories about community temporal dynamics; (iii) to examine the role of year-to-year variation in rainfall in vegetation dynamics; and (iv) to determine the factors causing site-to-site differences in response. In addition to temporal dynamics, spatial variation in vegetation recovery and the environmental factors that influenced it were also of interest. At each site, the variables measured annually included herbaceous cover (as a percentage of ground surface) and plant species richness. Time-invariant spatial covariates that were measured included pre-fire age of the shrub stand (estimated from ring counts of stem samples), fire severity (based on skeletal remains of shrubs), and site abiotic characteristics.

One objective of the repeated sampling was to examine the dynamics of plant biodiversity so as to better understand the role of disturbance in maintaining diversity on the landscape. Previous experience suggested that immediately following fire there is a

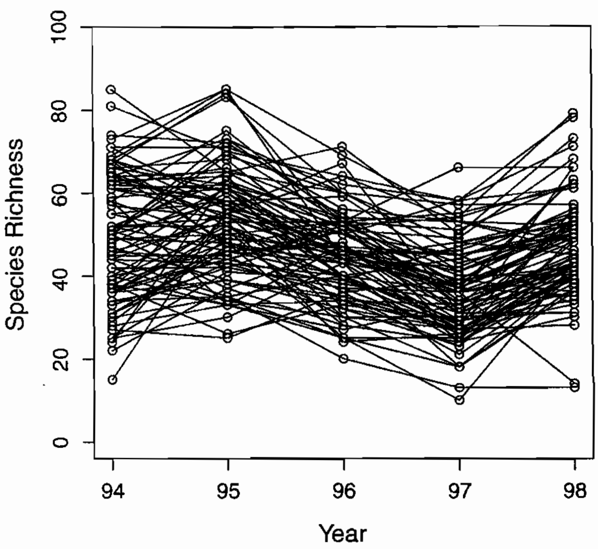

Figure 15.5 Observed values of herb species richness for the 88 plots in the example data set from southern California shrublands.

flush of plant germination resulting in peak diversity in the first year. Over time, diversity is expected to decline due to reduced germination and as competition intensifies. This "pulse-response" hypothesis was examined in an earlier analysis by Grace and Keeley (2006) that used latent trajectory modeling and SEM to study diversity responses to the fires. Here we examine the temporal dynamics of these data in greater detail to illustrate SEM methods.

The temporal dynamics of plant species richness in the 88 plots with complete data represents a collection of trajectories. We begin by considering the individual trajectories (Fig. 15.5) and some of their characteristics (Fig. 15.6). Richness showed a general decline over time, though with substantial fluctuations (Fig. 15.5a). A summary of the individual regression slopes (Fig. 15.6b) shows that the majority were negative. The raw data showed higher than average richness in 1995 and 1998 and it was observed that mean annual precipitation was higher than normal in those years (Fig. 15.6c). Once precipitation is considered, the decline over time is more conspicuous (Fig. 15.6d).

The SEM analyses presented in the following sections were conducted using the software Mplus (Muthén and Muthén 1998–2010). The data and software code for the final model are available as on online supplement to our chapter. There exist numerous software platforms for the analysis of SE models. Some of the more popular ones include Amos, LISREL, EQS, and Mplus. In addition, there are modules in the R software for SEM (e.g. "sem" and "lavaan"). All of the packages mentioned have the capabilities needed to analyze the models below, although their other capabilities and their ease of use differ.

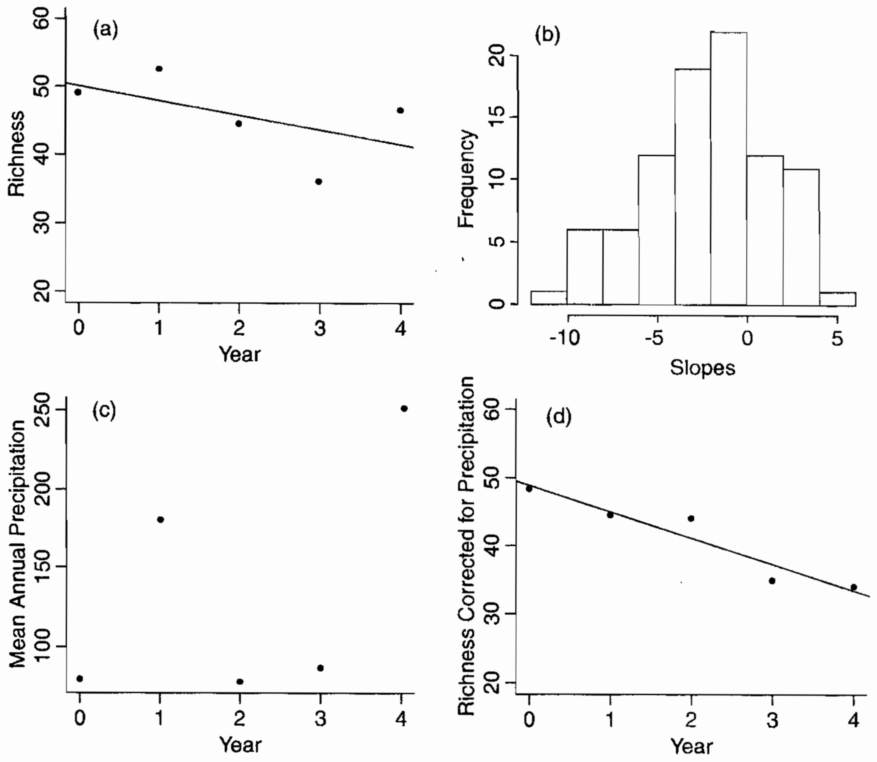

Figure 15.6 Some characteristics of the data being modeled in our extended example. Years 0–4 in the graphs correspond to 1994–1998: (a) mean richness over time, (b) histogram of individual slopes for the 88 trajectories, (c) mean annual precipitation values, and (d) plot of mean richness corrected for mean annual precipitation.

A linear latent trajectory model (LTM)

For didactic purposes, we begin by modeling the richness data using a simple, linear LTM [Equations (15.8)–(15.10) and (15.13)]. With this model we are fitting a linear relationship to the trajectories in Fig. 15.5 and seeking estimates of the means and variances for the slopes and intercepts of the individual cases (Fig. 15.7). Illustrated in the latter figure are some common characteristics of the LTM, such as the fact that the loadings connecting the intercept with the observed richness values from each time are all fixed at 1.0. The loadings connecting the slope variable with the observed richness values progress from 0 to 4 in increments of 1. As described earlier, this implementation is one where each individual trajectory is described by an intercept and a slope and the intercepts and slopes have a distribution of values across the various cases.

There are a total of five observed variables involved in this analysis; therefore, the number of known pieces of information includes their five means + their five variances + their 10 covariances, which equals a total of 20. Of the 25 parameters in the model (Table 15.1), 15 parameters have fixed values so only 10 parameters need to be

Table 15.1 Summary of parameters in simple linear LTM (Fig. 15.7).

Estimate type	Loadings[1]	Covariances	Variances	Means	Intercepts	Total
fixed value	15	0	0	0	0	15
estimated	0	1	7	2	0	10
total	15	1	7	2	0	25

[1] The 15 fixed loadings include the five loadings from the intercept to the five responses (all fixed to a value of 1), the five loadings from the slope to the five responses (fixed to values 0–4), and the five predicted intercepts for the responses r94–r98, which were all fixed to 0.

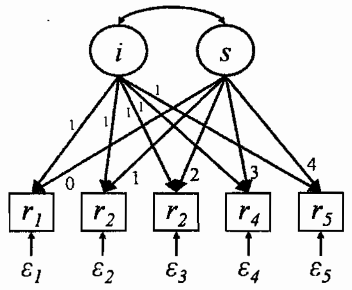

Figure 15.7 Linear latent trajectory model for species richness r_1–r_5 in the example. Summary of parameters from this model is given in Table 15.1 and results are presented in Table 15.2.

estimated (Table 15.2). This leaves $20 - 10 = 10$ model degrees of freedom for hypothesis testing.

Can a simple linear LTM provide an adequate representation of the data? Based on the large degree of discrepancy between observed and model-implied covariances and means (a model chi-square = 162.4 with 10 model degrees of freedom and *P*-value < 0.001), we conclude that the model is a very poor representation of the forces that shaped these data. Ignoring this important point for the time being, we can see from Table 15.2 that our estimate of the mean intercept for the 88 time trajectories is 50.94. The calculated standard deviation for the random intercepts equals 8.79 (the square root of the variance of the intercepts, 77.33). The estimated mean slope in this case is –2.66 and its variance is effectively 0. Assuming these values to be valid estimates, it is interesting that the variance of the slopes is so small, implying a great deal of uniformity in the average declines in richness over time in the different plots. Overall, the estimated intercept and slope derived from our LTM are generally consistent with the mean response for our trajectories (Fig. 15.6a). However, we have reason to suspect that our model does not capture the important factors controlling richness dynamics because of the poor model fit.

Do we have homogeneous variances over the time trajectory? While it is not mandatory in the SEMs, we wish to know whether we can represent our mean time trajectory more simply with a common error variance across the years. It is possible to estimate a single error variance for the trajectory and to test for whether there is a common error variance. To accomplish this we can estimate a single value for all ε_1–ε_5. When we

Table 15.2 Select results for the simple, linear latent trajectory model (Fig. 15.7). The codes "r94–98" refer to species richness per plot during the years 1994 through 1998. Number of cases (plots) in the data set = 88. The model chi-square was 162.4 with 10 model degrees of freedom and associated *P*-value < 0.001, all of which indicate very poor model-data fit.

Parameters[1]	Estimate	Std. error	Critical ratio	*P*-value
slope → r94	0 (fixed)			
slope → r95	1.0 (fixed)			
slope → r96	2.0 (fixed)			
slope → r97	3.0 (fixed)			
slope → r98	4.0 (fixed)			
mean of intercept, *i*	50.94	1.50	34.01	< 0.001
mean of slope, *s*	–2.66	0.58	–4.61	< 0.001
covariance between *i* and *s*	5.53	7.23	0.76	0.44
VAR(intercept)	77.33	26.95	2.87	0.004
VAR(slope)	–4.01[2]	2.65	–1.52	0.13
R^2 for r94	0.28			
R^2 for r95	0.51			
R^2 for r96	0.67			
R^2 for r97	0.44			
R^2 for r98	0.32			
Average R^2	0.44			

[1] Note that for the *P*-values associated with parameters, in contrast to those associated with the model chi-square, significant parameters have *P*-values LESS THAN 0.05.
[2] Note that the model-implied variance for the slope estimate was negative. A Wald test shows that the value of –4.014 was not significantly different from a value of zero (note associated *P*-value of 0.130). While a value of 0 for this variance estimate is technically acceptable, if our variance was significantly negative, that would be considered an inadmissible estimate that suggests model misspecification.

do this, we get an estimate of 90.60. However, our model chi-square increases from 162.4 to 183.9 (21.5 points) while releasing four degrees of freedom because four fewer parameters were estimated. This magnitude of chi-square increase indicates that a single error variance for the trajectory is an oversimplification for our data.

An LTM with time-varying covariates

What role does annual variation in precipitation play in year-to-year variations in richness? In addition to the common effects of trajectory intercept and slope, we have reason to believe that conditions that affect richness in any given year vary over time. One conspicuous time-varying feature that seems to correlate with the deviations of observed richness from a simple trajectory is annual precipitation. This covariate is time-varying (different values for each time) and thus our model corresponds to that described by Equation (15.19). However, our precipitation covariate in this case is made up of single study area-wide values, one for each year for all 88 cases, and is therefore a fixed rather than random effect. Figure 15.8 illustrates the form of the model in which we have allowed annual richness values to be a function of precipitation. Further, because

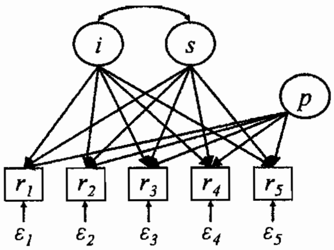

Figure 15.8 Latent trajectory model for species richness r_1–r_5 with covariate p, for precipitation, included. Results are summarized in Table 15.3.

our regional rainfall values are very approximate for our specific locations, we have used fixed loadings to indicate that 1995 was a wetter than average year and 1998 was much wetter than average (Fig. 15.6c), allowing "normal" years to have loadings of 0, the wet year a loading of 1, and the very wet year a loading of 2. Typically, we would allow the precipitation covariate to correlate with our latent intercept and slope. However, since precipitation has only one unique value for each year, we assigned it a very small fixed variance of 0.01 and fixed its covariance with other exogenous variables to zero to simplify the assumptions made in model. For this model, we obtained a chi-square of 43.9 with nine degrees of freedom and $P = 0.001$. We consider this model to have inadequate fit and in need of further modification before it can be used to draw interpretations.

One thing we noticed in looking at the results was a residual correlation between richness in 1994 and in 1995. This suggests the errors may be correlated over time, perhaps due to autocorrelation, which would be expected in these data. Including an error correlation in our model, we obtain a chi-square of 33.05 with eight degrees of freedom (a drop of 10.85). We retain the error correlation in our model for the time being, though the introduction of additional covariates could conceivably make it disappear. At this point, our model still does not fit the data very well and we continue examining our question about the role of precipitation in the next section where we allow precipitation to have a nonlinear effect.

An LTM with a nonlinear effect from the time-varying covariates

Examination of residuals from the previous model suggested that richness in 1996 was somewhat greater than our model predicted (see Fig. 15.6d for an approximate view). If theoretically justifiable, one way we could model this is to freely estimate the loading from precipitation for 1996, along the lines of what is represented in Equation (15.14). If we relax the constraint of a fixed loading from precipitation for that year, model fit is substantially improved, providing empirical (but not theoretical) justification for fitting a nonlinear relationship. A model allowing for the free estimation of a precipitation effect for 1996 explains the mean and covariance structure in the data much better (chi-square $= 13.401$ with seven degrees of freedom and a P-value $= 0.063$). Select results are presented in Table 15.3. The estimated value for the loading for 1996 using this

Table 15.3 Select results from nonlinear latent trajectory model (Fig. 15.8) with precipitation as time-varying covariate included. Model chi-square = 13.401 with seven degrees of freedom and a P-value = 0.063.

Parameters[1]	Estimate	Std. error	Critical ratio	P-value
slope → r94	0 (fixed)			
slope → r95	1.0 (fixed)			
slope → r96	2.0 (fixed)			
slope → r97	3.0 (fixed)			
slope → r98	4.0 (fixed)			
p → r94	0 (fixed)			
p → r95	1 (fixed)			
p → r96	0.550	0.113	4.851	< 0.001
p → r97	0 (fixed)			
p → r98	2 (fixed)			
mean of intercept, i	49.286	1.423	34.629	< 0.001
mean of slope, s	–4.352	0.344	–12.668	< 0.001
mean of precip, p	7.489	0.508	14.752	< 0.001
covariance between i and s	5.995	6.314	0.949	0.342
VAR(intercept)	76.30	27.34	2.790	0.005
VAR(slope)[1]	–2.353	1.988	–1.184	0.237
R^2 for r94	0.281			
R^2 for r95	0.577			
R^2 for r96	0.683			
R^2 for r97	0.752			
R^2 for r98	0.554			
Average R^2	0.569			

[1] Note again that the model-implied variance for the slope estimate was negative, but not significantly different from a value of zero. We conclude once again that the variance of the slope is approximately zero.

nonlinear approach is 0.55, which implies that conditions were favorable for richness in that year, but not as favorable as in 1995. It seems that this favorability for 1996 was not due to differences in total rainfall per se, so we speculate that perhaps spring temperatures, rainfall distribution, or some other particular set of circumstances favored richness in that year. We will revisit this issue later once we have illustrated more modeling possibilities.

The estimated intercept based on our model that includes precipitation is 49.29 species per plot, which is very close to the estimate from our simpler model (Table 15.2). However, our new estimate of slope is –4.35 (Table 15.3), indicating a steeper decline than estimated from our model that ignored precipitation (where the slope was –2.656). The reason the intercept is much more negative in our model including precipitation is because the elevated richness values in 1995 and 1998 are now being explained by increased precipitation in those years, which implies that if precipitation levels in those years had been comparable to the other years (1994, 1996, and 1997), we would have expected a loss of 4.35 species each year across time (for a total loss of 17.4 species

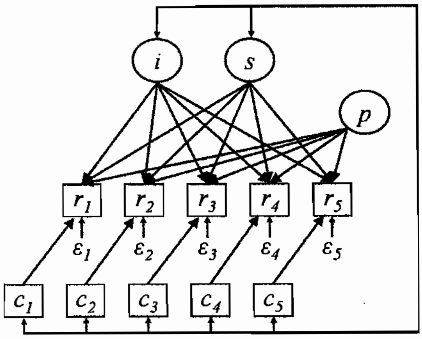

Figure 15.9 Latent trajectory model for species richness r_1–r_5 with covariate p, for precipitation, and covariates c_1–c_5, herb cover, included. Results are summarized in Table 15.4.

or 35% of the initial species during the 5-year period). In addition to the fact that our mean precipitation effect is significant, we can also see that our R^2 values for the individual richness responses each year have increased for the years 1995–1998. Average R^2 increased from 0.442 for our model without precipitation included (Table 15.2) to an average of 0.569 (Table 15.3). Taken together, these pieces of evidence support the interpretation that precipitation has a very powerful influence on richness in any given year. Once we control for annual variations in precipitation, we see there is a steady decline in richness following fire as predicted by theory.

Are annual variations in herb cover predictive of year-to-year variations in richness? It is useful for illustrative purposes to consider a more typical time-varying covariate that possesses individual values for each plot at each time of measurement. For this reason, we include in our model year-to-year variations in herb cover. There are actually several different ways we could include the herb-cover variables in the overall model of richness, two of which we will describe here (the choices described here are for purposes of illustrating SEM modeling methods and not a comprehensive evaluation of the possibilities). First, it would be possible to expand our latent trajectory structure to model a time trajectory for herb cover variations as we did with richness (using the architecture in Fig. 15.7, but with cover as the response variable). This would cause our total model to contain two interacting trajectories, along with the effect of precipitation. A further discussion of some of the possibilities for such models can be found in Bollen and Curran (2006, chapter 7).

Second, it would be possible to ignore the causal forces driving annual variations in herb cover and treat it as a simple time-varying covariate. This is the approach we have taken to address the question of whether cover variations help explain our richness trajectory, as well as to test whether there remains a monotonic decline in richness once we control for herb-cover temporal variations (Fig. 15.9). Initial results indicated the need to include a correlation between herb cover and richness for 1994. We again interpret this correlation as indicative of autocorrelation in the data, which is common for spatially arranged data such as these.

The inclusion of herb cover as a time-varying covariate of richness in the model raises the question of what kind of linkage between herb cover and richness we should

Table 15.4 Select results from nonlinear latent trajectory model (Fig. 15.9) with precipitation and herb cover as time-varying covariates. Model chi-square = 22.096 with 19 degrees of freedom and a P-value = 0.280. Note that a common parameter estimate describes the effect of herb cover on richness for 1995–1998.

Parameters	Estimate	Std. error	Critical ratio	P-value
slope → r94	0 (fixed)			
slope → r95	1.0 (fixed)			
slope → r96	2.0 (fixed)			
slope → r97	3.0 (fixed)			
slope → r98	4.0 (fixed)			
p → r94	0 (fixed)			
p → r95	1 (fixed)			
p → r96	0.596	0.115	5.183	< 0.001
p → r97	0 (fixed)			
p → r98	2 (fixed)			
herb cover 94 → r94	0.046	0.027	1.723	0.085
herb cover 95 → r95	0.035	0.012	2.978	0.003
herb cover 96 → r96	0.035	0.012	2.978	0.003
herb cover 97 → r97	0.035	0.012	2.978	0.003
herb cover 98 → r98	0.035	0.012	2.978	0.003
mean of intercept, i	45.832	1.888	24.282	< 0.001
mean of slope, s	–4.728	0.373	–12.69	< 0.001
mean of precip, p	7.343	0.523	14.04	< 0.001
covariance between i and s	7.029	5.823	0.207	0.227
VAR(intercept)	71.38	25.49	2.800	0.005
VAR(slope)	–2.374	1.851	–1.282	0.200
R^2 for r94	0.326			
R^2 for r95	0.610			
R^2 for r96	0.699			
R^2 for r97	0.749			
R^2 for r98	0.575			
Average R^2	0.592			

specify. We might theorize that there should be a consistent dependence of richness on cover over time, in which case we wish to estimate a single coefficient. What was found was that the dependence of richness on cover for 1995 through 1998 could indeed be represented by a single coefficient without a loss of information (i.e. including this constraint did not significantly increase model chi-square). However, the dependence of richness on cover in 1994 was of a different magnitude from the other years (Table 15.4). Collectively, the results from this analysis indicate that variation in herb cover is a significant time-varying covariate for our richness trajectory.

An LTM with time-varying and time-invariant covariates

How do spatial, time-invariant covariates play a role in our model? The earlier analysis of these data by Grace and Keeley (2006) illustrated that richness variations among sites

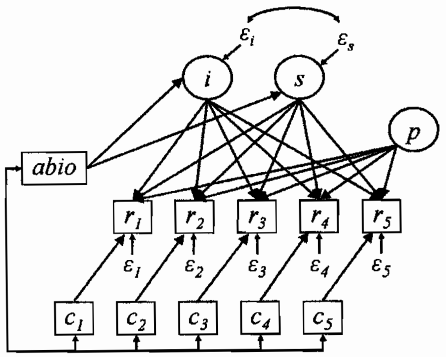

Figure 15.10 Latent trajectory model for species richness r_1–r_5 with time-varying covariates p, precipitation, and c_1–c_5, herb cover, as well as time-invariant covariate, *abio* (abiotic conditions). Results are summarized in Table 15.5.

in the first year following fire depend on spatial variations in environmental conditions. To illustrate further the integrative framework of SEM, we show how the effects of time-invariant covariates can be modeled. To illustrate the flexibility of the SEM methodology, we include a network of three time-invariant covariates: abiotic favorability (a moderating factor that potentially alters responses), age of stand that burned (a primary predictor of system response to fire), and fire severity (a hypothesized mediator of the stand-age impacts).

Regarding abiotic favorability, in the earlier study it was observed that sites (with one plot at each site) differed in their abiotic soil conditions and that some sites were more favorable for species richness compared to others. Thus, we might expect that variations in the intercepts for our individual trajectories can be related to the site-to-site variations in those soil conditions. Concomitant with differences in intercepts, rates of decline (slopes) might also vary among sites depending on abiotic favorability. Regarding stand age and fire severity, earlier work (Grace and Keeley 2006) demonstrated that in the first year after fire (1994) older stands experienced higher fire severities, reduced plant growth, and (indirectly) reduced richness.

The model specifying the effects of abiotic conditions, stand age, and fire severity is shown in Fig. 15.10. In this model we are able to control for variations in abiotic favorability while (i) testing the hypothesis that stand-age effects on richness are mediated by effects on fire severity and (ii) partitioning the direct and indirect (through herb cover) effects of fire severity on post-fire richness.

Results are summarized in Table 15.5. Model fit was good, with a chi-square = 44.88 with 38 degrees of freedom and a P-value = 0.205. However, in this model, we continue to allow the precipitation effect on richness to be modeled with free parameters. Specifically, we permit an empirically determined nonlinear response by richness in 1996 to precipitation. As mentioned earlier, we have no theoretical justification for such a nonparametric nonlinear effect, though up to this point in the chapter we have not had an alternative modeling option. In the next section, we illustrate the inclusion of an autoregressive effect in an LTM, which allows us a more meaningful way to explain the

Table 15.5 Select results from latent trajectory model with time-varying and time-invariant covariates (Fig. 15.10). Path from abio to slope was found to be non-significant and set to zero. Model chi-square = 44.88 with 38 degrees of freedom and a *P*-value = 0.205. Note that a common parameter estimate describes the effect of herb cover on richness for 1995–1998. Paths involving time-invariant covariates shown in bold.

Parameters[a]	Estimate	Std. error	Critical ratio	*P*-value
slope → r94	0 (fixed)			
slope → r95	1.0 (fixed)			
slope → r96	2.0 (fixed)			
slope → r97	3.0 (fixed)			
slope → r98	4.0 (fixed)			
p → r94	0 (fixed)			
p → r95	1 (fixed)			
p → r96	0.601	0.117	5.141	< 0.001
p → r97	0 (fixed)			
p → r98	2 (fixed)			
herb cover 94 → r94	0.155	0.026	5.970	< 0.001
herb cover 95 → r95	0.042	0.015	2.833	0.005
herb cover 96 → r96	0.042	0.015	2.833	0.005
herb cover 97 → r97	0.042	0.015	2.833	0.005
herb cover 98 → r98	0.042	0.015	2.833	0.005
fire -> r94	**–1.544**	**0.425**	**–3.631**	**< 0.001**
fire -> herb cover94	**–8.420**	**1.804**	**–4.666**	**< 0.001**
age -> fire	**0.061**	**0.013**	**4.841**	**< 0.001**
abio → intercept, *i*	**0.594**	**0.235**	**2.531**	**0.011**
intercept for intercept[a], *i*	16.222	11.79	1.376	0.169
mean of slope, *s*	–4.800	0.369	–13.02	< 0.001
mean of precip, *p*	7.308	0.526	13.90	< 0.001
mean for abio (fixed est)	48.93	–	–	–
covariance between *i* and *s*	3.218	5.657	0.569	0.569
VAR(intercept error)	83.33	25.33	3.290	0.001
VAR(slope)[b]	–2.328	1.828	–1.273	0.203
R^2 for r94	0.25			
R^2 for r95	0.67			
R^2 for r96	0.72			
R^2 for r97	0.73			
R^2 for r98	0.56			
Average R^2 for richness	0.59			
R^2 for herb cover94	0.20			
R^2 for fire	0.21			

[a] Since the LTM intercept latent variable is endogenous in this model, it has an estimated intercept based on its regression on abiotic conditions. To obtain an estimate of the intercept of the latent trajectory in this case, we calculate $16.222 + 48.93 \times 0.595 = 45.35$.

[b] Note that the model-implied variance for the slope estimate was negative. A Wald test showed that the value of –2.328 was not significantly different from a value of zero.

higher-than-expected richness in 1996. Therefore, we will reserve interpretive statements for the next, final model presented.

An autoregressive latent trajectory model (ALT)

As mentioned earlier, an alternative modeling approach is to explain temporal change using an autoregressive approach [Equation (15.7)]. With this approach, we model richness at each time as some function of richness the previous time as well as effects from covariates. To a degree, the latent trajectory and autoregressive models are two different ways of representing the same set of causal forces. In the latent trajectory model, we hypothesize a common factor controlling change over time. For example, if richness is stimulated by fire-induced release of species from the seed bank and the decline over time is a progressive loss of fire-dependent species, a latent trajectory captures this causal process. On the other hand, if richness in each year is dependent on the number of species the previous year because "diversity begets diversity" through seed production and plant recruitment in the following year, then there is a direct effect between richness in one year and richness in the next year. Of course, it is possible that both kinds of processes operate simultaneously, which means an autoregressive path could be included within a latent trajectory model. Here we address a question we ignored earlier about year-to-year effects of richness.

Does elevated richness in one year beget more richness in the next year, all other factors equal? We noticed, as mentioned earlier, that richness in 1996 was higher than expected based on a simple consideration of year-to-year variations in precipitation. Earlier we hypothesized that perhaps this was because weather conditions were favorable in that year in some way other than captured by the annual precipitation. To incorporate this effect, we allowed the loading of precipitation on richness in 1996 to be freely estimated.

An alternative mechanism to explain the elevated richness observed in 1996 is that the high levels of richness in 1995 were promoted by higher than normal precipitation that carried over to 1996. This year-to-year effect can be evaluated by modifying our latent trajectory model to (i) force the precipitation effect to be linear and then (ii) include an autoregressive relationship between richness in 1995 and 1996 (as shown in Fig. 15.11). To accomplish these two things, along with adding a path, we set the loading from precipitation to richness in 1996 to 0, making our model linearly related to the annual precipitation data. Estimation of this model yielded a chi-square of 48.68, with 38 df, and $P = 0.115$. We consider this model to have good fit and accept it as our final model in this demonstration of modeling options. Select results are presented in Table 15.6.

The autoregressive path from species richness in 1995 to richness in 1996 was found to be significant, with a raw path coefficient of 0.073 ($P < 0.001$). This result supports the conclusion that the elevated richness in 1996 can indeed be explained by a carryover effect from 1995, a wetter than average year. We estimate that there was about a 7% enhancement of richness in 1996 from richness in 1995 (coefficient for the path r95 $\rightarrow$ r96 was 0.073, which is measured in terms of species enhancement in 1996 per species

Table 15.6 Select results from autoregressive latent trajectory (ALT) model (Fig. 15.11). Path from abio to slope was found to be non-significant and set to zero. Model chi-square = 48.68 with 38 degrees of freedom and a P-value = 0.115. In this model, an effect of richness in 1995 on richness in 1996 was included and the effect of precipitation on richness in 1996 was fixed to a value of 0. The added autoregressive effect is shown in bold (other autoregressive parameters were not significant).

Parameters	Estimate	Std. error	Critical ratio	P-value
slope → r94	0 (fixed)			
slope → r95	1.0 (fixed)			
slope → r96	2.0 (fixed)			
slope → r97	3.0 (fixed)			
slope → r98	4.0 (fixed)			
p → r94	0 (fixed)			
p → r95	1 (fixed)			
p → r96	0 (fixed)			
p → r97	0 (fixed)			
p → r98	2 (fixed)			
herb cover 94 → r94	0.152	0.026	5.867	< 0.001
herb cover 95 → r95	0.037	0.015	2.519	0.012
herb cover 96 → r96	0.037	0.015	2.519	0.012
herb cover 97 → r97	0.037	0.015	2.519	0.012
herb cover 98 → r98	0.037	0.015	2.519	0.012
r95 → r96	**0.073**	**0.017**	**4.210**	**< 0.001**
fire -> r94	−1.625	0.424	−3.834	< 0.001
fire -> herb cover94	−8.434	1.803	−4.677	< 0.001
age -> fire	0.061	0.013	4.841	< 0.001
abio → intercept, i	0.607	0.228	2.662	0.008
intercept for intercept[a], i	16.338	11.428	1.430	0.153
mean of slope, s	−4.803	0.372	−12.92	< 0.001
mean of precip, p	7.200	0.532	13.54	< 0.001
mean for abio (fixed est)	48.93	–	–	–
covariance between i and s	6.465	5.789	1.117	0.264
VAR(intercept error)	68.34	25.21	2.711	0.007
VAR(slope)[b]	−3.181	1.886	−1.686	0.092
R^2 for r94	0.19			
R^2 for r95	0.63			
R^2 for r96	0.72			
R^2 for r97	0.72			
R^2 for r98	0.55			
Average R^2 for richness	0.56			
R^2 for herb cover94	0.20			
R^2 for fire	0.21			

[a] Since the LTM intercept latent variable is endogenous in this model, it has an estimated intercept based on its regression on abiotic conditions. To obtain an estimate of the intercept of the latent trajectory in this case, we calculate $16.338 + 48.93 \times 0.615 = 45.212$, where 48.93 is the mean for the abio variable.

[b] Note that the model-implied variance for the slope estimate was negative, but not significantly different from a value of zero.

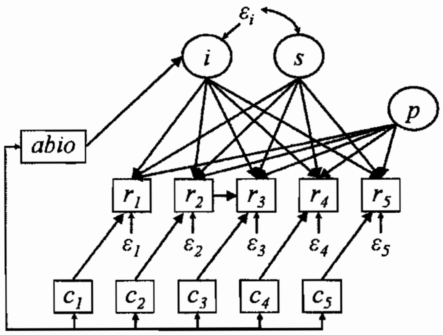

Figure 15.11 Autoregressive latent trajectory model for species richness r_1–r_5 with time-varying covariates p and c_1–c_5, as well as time-invariant covariate, *abio* (abiotic conditions) included. Only a single autoregressive effect $r_2 \rightarrow r_3$ was found to be significant. Results are summarized in Table 15.6.

in 1995). This year-to-year enhancement occurred at the same time that there was a general decline in richness following fire of 4.80 species per year (nearly a 50% total decline over the 5-year period).

In addition to the unstandardized results summarized in Table 15.6, we can also calculate standardized coefficients, which provide an additional way of looking at results. A complete set of results, including the standardized coefficients for indirect and total effects, can be found along with the data and Mplus code at the online supplement. Here we present a few summary findings for the effects of time-invariant covariates; specifically, we present the standardized direct, indirect, and total effects of stand age ("age") on spatial variation in species richness following fire "r_1" (richness in 1994).

First, the results confirm that age had no direct effect on richness in 1994 by comparing models with and without a direct path from age to richness. Inclusion of a path from age to r_1 in Fig. 15.11 leads to a decrease in model chi-square of only 0.46, well below the critical cutoff chi-square difference value of 3.84. This result supports the earlier interpretation (Grace and Keeley 2006) that the reason older stands that burned had lower richness was because of higher fuel loads and higher fire severities in those older stands. Second, we estimate that the standardized total effect of age on richness in 1994 = –0.153. This standardized total effect can be assembled from four standardized path coefficients: *age* → *fire* = 0.459, *fire* → c_1 = –0.443, c_1 → r_1 = 0.333, and *fire* → r_1 = –0.187. The indirect pathway *age* → *fire* → r_1 has a standardized effect of (0.459)(–0.187) = –0.085, while the other indirect pathway *age* → *fire* → c_1 → r_1 has a standardized effect of (0.459)(–0.443)(0.333) = –0.068. Together these add up to the total effect of age on richness of –0.153.

What is perhaps the most biologically interesting effect in this web of relationships is that fire has two different kinds of negative influence on richness, one through a suppression of plant cover following fire in high severity plots (*fire* → c_1 → r_1 = –0.148) and an additional reduction in richness that is independent of cover (*fire* → *r*– = –0.187). These two effects appear to be of roughly equal strength and combine to create a moderately strong overall effect (–0.335).

Future research and development

The SEM framework provides very substantial flexibility in model specification. The possibilities for modeling longitudinal data are extensive and we have only touched on a few of the more basic approaches. The interested reader is encouraged to examine Bollen and Curran (2006), which additionally considers (i) further approaches to modeling nonlinear trajectories, including oscillating dynamics; (ii) comparing trajectories among groups; (iii) mixture models in which group identity is initially unknown; (iv) models involving dichotomous and ordinal responses; (v) ways of dealing with measurement error; and (vi) various ways of dealing with missing data. The use of Bayesian methods in SEM potentially further increases our flexibility in specifying and solving models.

Along with the great variety of options for modeling within the SEM framework is an attendant complexity that can challenge those who wish to learn to use these methods. What we believe is most needed at this point to advance use of these methods is the development of documented examples and decision-support systems that can guide and aid model development and testing. Ecological data analysts encounter numerous complexities, such as spatial autocorrelation, nonlinearities, and hierarchical structures that can make the modeling of longitudinal data complex. This is coupled with the fact that those from the biometric tradition face a substantial learning curve to become proficient with SEM because it is not part of our usual training. Focus here needs to be not only on statistical issues but how to address the many interesting ecological questions that could be examined with these methods.

A somewhat separate area where further work is needed is with the deductive process of projecting the consequences of model results. Particularly deserving of further exploration is the interface between SEM and predictive probabilistic networks (Cowell *et al.* 2007). Both analysis traditions can be classified as forms of graphical modeling, but probabilistic networks (sometimes referred to as Bayesian networks) are specifically designed for forecasting and informing decisions in the face of uncertainty. Ultimately, society wants not only to know how systems work but also the likelihood of their exhibiting various states in the future. This extrapolative process requires us to apply the network structures and parameter estimates we discover with SEM to projections to new situations, something imminently possible, but infrequently considered.

Summary

There are many questions that can be asked of temporal data and the underlying causal forces that structure it. In the SEM tradition, we seek models that test the plausibility of underlying causal forces hypothesized in our models. We also seek to understand the interworkings of multiple processes that simultaneously influence system dynamics. A premium is placed on interpretability in SE models and the focus is rarely on simply arriving at a set of predictors. Because SEM is concerned with underlying causes, which can only rarely be measured directly, latent variables are often incorporated in such models. It is entirely possible to model temporal dynamics using SEM methods without latent variables. Autoregressive models, for example, allow for the examination of causal networks without latent variables. Latent variables in SE models permit a greater degree of abstraction and associated generality, however. In this chapter we emphasize the latent trajectory model (LTM) which can represent the characteristics of a set of repeated measures in a simple and elegant way. Such models seek to describe trajectories with a minimum of parameters, while allowing for complexities to be included as needed.

In the ecological example explored in this chapter, which deals with vegetation dynamics in a fire-prone landscape, we use LTMs to ask whether there is a decline in species richness over time (and how rapid this decline is) once the influences of time-varying covariates are controlled. The pulse-decline pattern studied in this example represents the influence of periodic fire on a fire-adapted community where occasional post-fire recruitment from the seedbank affects richness. The LTM allows us to not only evaluate the hypothesis of a post-fire decline and to examine factors that cause short-term deviations from that decline, but also to examine forces controlling the variations among individual plots within the population. The inclusion of autoregressive effects within the general trajectory model further permits us to determine that wet conditions in a year not only enhance diversity in that year but can have a carryover effect on the following year.

Overall, SEM provides a flexible framework for learning from monitoring data. It is not designed for purely descriptive analyses, but instead, works with models having some theoretical justification. A great variety of variations are possible and their use depends on the hypothesized mechanistic processes and the characteristics of the data. An immediate need is a greater illustration of the many possible uses of SEM with ecological data.

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Design and Analysis of Long-term Ecological Monitoring Studies

s and methods to natural resource managers, scientists, and students. The chapter

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